Rated MPG for Confusion: Using Gas Mileage to Learn Graphing and Data Analysis

by

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Part I – A Drop in the Bucket.

Cody's 16th birthday is this Friday. He's psyched because he's finally getting his unrestricted driver's license. His parents have decided to let him drive to school instead of having to take the school bus.

Cody's parents, Mr. and Mrs. Andrews, have two cars—a Toyota Corolla and a Jeep Grand Cherokee SUV. They use the Corolla for driving to work and running errands around town. It is a very economical car and gets 28 miles per gallon (mpg) around the city. However, Mr. and Mrs. Andrews need the larger SUV to haul around their two big St. Bernard dogs. The Jeep Grand Cherokee SUV gets 13 mpg around the city.

The problem is that the price of gas keeps going up. To save money, the Andrews have decided to trade one of their cars for a more fuel-efficient vehicle. They have come up with two options:

1. Keep the SUV and trade in the Corolla for a Toyota Prius Hybrid (the Prius makes 48 mpg in the city); or
2. Keep the Corolla and trade the SUV for a Minivan (they have their eyes on either a Toyota Sienna or Honda Odyssey; both of these models make 17 mpg city driving).

Put yourself in the place of Cody's parents. You have two possible decisions. You could keep the SUV and trade in the Corolla for the Hybrid. Going from 28 (Corolla) to 48 (Hybrid) miles per gallon (mpg) should really save money on gasoline. That is a change of 20 mpg! Or, you could trade the SUV for the Minivan and keep the Corolla. The SUV gets about 13 mpg and the Minivan achieves slightly better gas mileage at 17 mpg. That is a change of only 4 mpg!

Questions

1. Without doing any calculations, what car do you think Cody's parents should trade in? Should they keep the SUV and trade the Corolla for the Hybrid or should they keep the Corolla and trade the SUV for the Minivan? Offer a hypothesis on the trade that will save them the most money in gasoline. Offer an explanation for your hypothesis.
2. If you were Cody's parents, what other factors might affect your decision about what type of car you would let Cody drive?

Part II – Gas Guzzler

Cody's parents decided to keep the SUV and trade in their Corolla for the Hybrid. They also decided to let Cody drive the SUV. "Happy Birthday, Son," said Mr. Andrews. "Here are the keys. Be careful!"

"Wow, thanks, Dad!" Cody took the keys and ran outside. The SUV was shining in the evening sunset light. He couldn't wait to take it for a spin around the block.

After a week of driving, Cody quickly realized that he got stuck with the gas guzzler. He wasn't making that much money at his crummy minimum wage job. He was starting to think that he might need to get a second job just to buy gas!

"Dad," Cody said, "I had no idea the SUV used so much gas. I'm only driving to work and school, but it's costing me a fortune."

"Don't worry," said Mr. Andrews. "Your mother and I will give you the money that we're saving on gas with our Hybrid to help fill up your tank."

Questions

Let's take our four cars and say we drive them for 100 miles. Now answer the following questions.

1. First, calculate the number of gallons needed to drive 100 miles in each car and place this value in the fourth column of Table 1. Then, project the cost if the gasoline is priced at $4 per gallon and place these costs in the last column. *Note:* Round all answers to the hundredths.

Table 1

Car	Distance (miles)	Miles per gallon (mpg)	Gallons of gas needed to go 100 miles	Cost (in $) to go 100 miles if gas costs $4 per gallon
SUV	100	13		
Minivan	100	17		
Corolla	100	28		
Hybrid	100	48		

2. Assuming that Cody's family needs one small car (Corolla or Hybrid) and one large car (SUV or Minivan), calculate the total cost of each combination possible in the table below.

Table 2

Car combination	Total cost (in $) to drive each car 100 miles
SUV-Corolla	
Minivan-Corolla	
SUV-Hybrid	
Minivan-Hybrid	

a. Which combination would be the most expensive for the family?

b. Which combination would be the most economical for the family?

c. As noted in Part I, Cody's parents currently have a Corolla and an SUV. What is the total cost (in $) to drive 100 miles in the vehicles that Cody's parents started with?

d. Also, as discussed in Part I, Cody's parents are planning to trade only one car. They can keep the Corolla and trade the SUV for a Minivan or they can keep the SUV and trade the Corolla for the Hybrid. Fill in Table 3 to help you decide which trade would save Cody's family the most money from what they have now.

Table 3

Cost to drive 100 miles with the vehicles that Cody's parents have before the trade	Cost to drive 100 miles in the new combination of cars after the trade	Savings (difference between columns)
SUV & Corolla = $	Minivan & Corolla = $	$
SUV & Corolla = $	SUV & Hybrid = $	$

3. Go back to Part I and look at the hypothesis you constructed in Question 1. You were asked to formulate a hypothesis based on which vehicle Cody's parents should keep and which vehicle they should trade in terms of saving the most money on gasoline.

   a. Do the calculations in Table 3 prove or disprove your hypothesis? Reflect about this and offer an explanation.

   b. Cody's parents chose to keep the SUV and trade in the Corolla for the Hybrid. Did they make the right decision in terms of saving money? Explain.

   c. Cody's father, Mr. Andrews, offered to help Cody pay for gas for the SUV: *"Don't worry," said Mr. Andrews. "Your mother and I will give you the money that we're saving on gas with our Hybrid to help fill up your tank."* Calculate how many gallons of gasoline Cody's parents were able to buy for Cody, assuming that gas is priced at $4 per gallon and the cars are driven 100 miles.

   d. How many miles would this amount of gasoline get Cody with the SUV making 13 mpg?

Part III – The Graph Explains All

What's going on here? Cody's parents had made what appeared to be the best decision. They traded in the Corolla to get the Hybrid, with an increase of 20 miles per gallon (mpg). However, the best choice seems to have been trading in the SUV for the Minivan with only a 4 mpg increase!

To really understand this conundrum, we have to look at the big picture. This means first looking at more data, graphing the data, and then obtaining the correct equation or function for this data. We looked at four cars and that was enough to answer a few simple questions. Looking at the big picture requires us to analyze more vehicles.

Table 4 lists 21 different vehicles with mpg ratings ranging from 9 to 48 mpg. Use this information to calculate the gallons of gas needed to drive the cars 100 miles.

Before you fill in the table, let's have the computer do what it's really good at—crunching numbers! See the directions after the table, as we are going to complete the rest of the activity on the computers.

Table 4

Car type and number	Distance (miles)	Miles per gallon (mpg)	Gallons needed to go 100 miles
1. Lamborghini Murcielago	100	9	
2. Chevy Avalanche 4WD SUV	100	11	
3. Cadillac Escalade SUV	100	12	
4. Dodge Viper Convertible	100	13	
5. Hummer H3 4WD SUV	100	14	
6. Chevy Corvette	100	15	
7. Porsche 911	100	16	
8. Toyota Sienna Minivan	100	17	
9. Mitshubishi Eclipse Spyder	100	19	
10. VW Beetle	100	20	
11. Dodge Avenger	100	21	
12. Honda Accord	100	22	
13. Nissan Altima	100	23	
14. Ford Focus	100	24	
15. Hyundai Elantra	100	25	
16. Minicooper	100	26	
17. Toyota Corolla	100	28	
18. Toyota Yaris	100	29	
19. Ford Escape Hybrid SUV	100	34	
20. Honda Civic Hybrid	100	40	
21. Toyota Prius	100	48	

Questions

1. Use a graphing program (such as Logger Pro 3 or Microsoft Excel) to calculate the gallons of gas needed to drive 100 miles (column 4 in Table 4 above). (For your convenience, a comma delimited version of Table 4, **gas_mileage_data.txt**, is available which can be imported into your program. For example, in Excel, go to Data→Import External Data→Import Data..., navigate to where you downloaded the file, and then walk through the wizard to identify the data as delimited, using commas as delimiters.)
2. Now we are ready to graph the data. For this graph, plot "miles per gallon" as the *x* variable and "gallons to go 100 miles" as the *y* variable. Print your graph.
3. Is your graph a straight line?
4. Describe the appearance of your graph. As one variable gets larger, what happens to the other variable?
5. Offer a suggestion for the type of graph (or function) shown.
6. Use your graphing program to determine the type of function or graph that best fits this data. At this point, do you need to modify your answer given in Question 5 above, yes or no? If so, modify it now.
7. Use your graphing program to generate the equation for this plot. The curve generated will follow the points as best as it can. What is the equation that describes your graph?
8. Your equation will have a constant of proportionality for this function. The constant of proportionality is often symbolized as either *A* or *k*.
   a. What is your constant of proportionality?
   b. We have seen this number before. Describe in words what this number means.
9. Your graphing program uses general variables, *x* and *y*. However, in our example, *x* and *y* stand for specific variables!
   a. What was our *x* variable (see horizontal label on the graph)?
   b. What was our *y* variable (see vertical label on the graph)?
   c. Now, rewrite the equation you wrote in Question 9 to match your set of variables.
   d. Verbalize the meaning of this graph in a sentence.
10. Now, use your graphing program to calculate the cost to drive 100 miles in each vehicle in Table 4. Label the new column "cost to drive 100 miles," and assume that gas costs $4 per gallon. The units will be "dollars."
11. Plot this new data with "miles per gallon" as the *x* variable and "cost to drive 100 miles" as the *y* variable.
12. Use your graphing program to determine the type of function or graph that best fits this data.
   a. What type (or function) is this graph?
   b. Write the equation using our variables.
13. What is the constant of proportionality for this graph?
14. What is the significance of this number (or how did the software come up with this number)?

15. Now let's go back to the original issue at hand. In Part I, you made a hypothesis for the choice of cars the Andrews family should trade in. There were two choices: Cody's family could keep the SUV and trade the Corolla for the Hybrid OR they could trade the SUV for a Minivan and keep the Corolla.

    In Table 1 of Part II, you calculated the gallons of gas needed to drive all four cars 100 miles and the cost to go this distance. Use your graphs to verify these data points.

16. It was noted earlier that it would have been a better choice economically for Cody's parents to trade in the 13 mpg SUV for the 17 mpg Minivan instead of trading in the 28 mpg Corolla for the 48 mpg Hybrid. Look at your "cost versus mpg" graph and offer an explanation for the reason for this.

17. Knowing the type of graph and its mathematical function helps to understand the relationship between cost and mpg rating. When asked to choose between trading in the 13 mpg SUV for the 17 mpg Minivan OR trading in the 28 mpg Corolla for the 48 mpg Hybrid, lots of people think the best decision economically would be to trade in the Corolla. All other variables being equal, this wouldn't be the best choice. Go home and ask your parents what they think about it. Why do you suppose most people choose the wrong answer?

Part IV – Straighten Up!

In this activity, you analyzed data and discovered several inverse relationships. Inverse relationships are not that hard to explain … as one variable increases, the other decreases in a predictable manner. However, they aren't as simple as direct relationships.

Questions

1. Why are direct relationships more simple and useful than indirect relationships?
2. We can transform an indirect or inverse relationship into a direct relationship by mathematically altering one of our columns of data. As an example, let's take the cost versus mpg graph (the last inverse graph) and turn it into a direct relationship (or straight line) graph.

   Using your graphing software, create a new column in the data table and call it "gallons per mile." Have the graphing software calculate the data for this column by taking the reciprocal of the mpg data (i.e., 1/mpg). This will transform your data into a direct relationship. Now graph the transformed data with "gallons per mile" on the *x*-axis and "cost to drive 100 miles" on the *y*-axis.

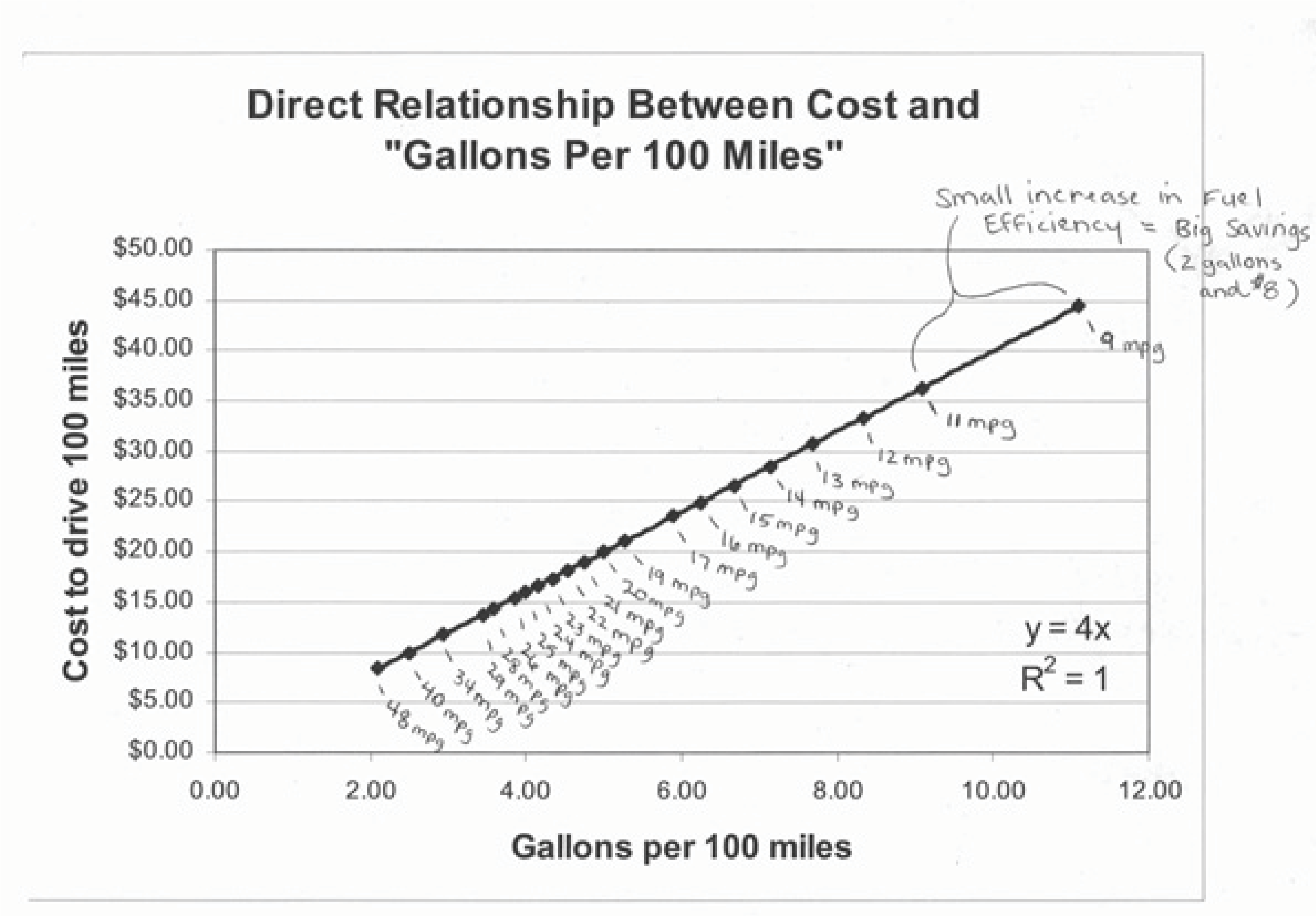
   Look at this new graph on the screen. Make sure to print it or show it to the teacher.

   Write the equation in $y = mx + b$ format. Use correct y and x variable names that match our data.
3. How is this equivalent to the equation of the inverse function?
4. What takes the place now of k, the constant of proportionality, that we saw in the inverse function (refer to Question 13 in Part III)?
5. Will the graph go through the origin? Please explain.
6. Now use your graphing software and data table to make one last graph. This time plot "gallons to go 100 miles" (which you calculated for Table 4 in Part III) on the *x*-axis and "cost to drive 100 miles" on the *y*-axis (still assuming gas costs $4 per gallon).

   Write the equation in $y = mx + b$ format. Use correct y and x variable names that match our data.
7. What takes the place now of the constant of proportionality? Where does this number come from?

   This final graph makes the *x*-axis easier to follow because it is round numbers instead of decimals. The graph shows a direct relationship between the cost and the number of gallons of gas used to drive 100 miles.

   This final graph also shows that small increases in mileage for the least fuel efficient vehicles results in large fuel and cost savings (see graph below). For example, going from 11 gallons per 100 miles (which corresponds to a mileage of 9 mpg) to 9 gallons per 100 miles (which corresponds to 11 mpg) gives large fuel and cost savings ($8). The same 2 mpg increase in fuel efficiency at the higher mileage rates gives smaller and smaller fuel and cost savings. For example, increasing from 26 mpg to 28 mpg corresponds to only saving 0.28 gallons of gas (3.85–3.57 gallons per 100 miles) and saving only $1 on gas costs. Obviously, the best scenario would be to only drive the most fuel efficient vehicles. Yet, small increases in fuel efficiency for the low mileage vehicles can still make a big difference in saving gas and money.



Direct Relationship Between Cost and "Gallons Per 100 Miles"
Cost to drive 100 miles
$50.00
$45.00
$40.00
$35.00
$30.00
$25.00
$20.00
$15.00
$10.00
$5.00
$0.00
0.00
2.00
4.00
6.00
8.00
10.00
12.00
Gallons per 100 miles
y = 4x
R^2 = 1
Small increase in Fuel Efficiency = Big Savings (2 gallons and $8)
9 mpg
11 mpg
12 mpg
13 mpg
14 mpg
15 mpg
16 mpg
17 mpg
19 mpg
20 mpg
21 mpg
22 mpg
23 mpg
24 mpg
34 mpg
40 mpg
48 mpg